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A TWO-FLUID MODEL FOR CRITICAL FLASHING FLOWS IN PIPES

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Abstract—A one-dimensional two-fluid model is described for the analysis of critical flows in pipes of nondiverging cross-sectional area. The model accounts for thermal nonequilibrium between the liquid and vapor bubbles and for interphase relative motion. Closure of the set of governing equations is performed with constitutive relationships which determine the pressure drop along the flow channel as a function of the flow regime, and the number and rate of growth of vapor bubbles in a variable temperature field in terms of the problem's primary state variables and geometrical configuration. An empirical correlation is derived which fits the number density of bubble nuclei in the flow as a function of the flow channel length-to-diameter ratio. Model predictions compare favorably with experimental data in small- and large-scale systems over a wide range of pressures and pipe diameters and lengths.

Key Words: two-phase flow, critical flow, two-fluid model, bubble growth, flashing, nonequilibrium

1. INTRODUCTION

The thermal hydraulic design and safety analysis of light water nuclear reactors involves the determination of the water conditions in the primary cooling system following a hypothetical rupture of a vessel or a pipe. Reliable estimation of the maximum flow (also known as critical or choked flow) and the fluid characteristics exiting from the ruptured flow circuit is a problem of fundamental importance in this case, as it determines the time at which the core is uncovered, and the forces produced by the expanding jet on the pipe and surrounding equipments in the vicinity of the jet. Physical understanding of the problem of two-phase critical flow is also useful in the design of boilers, refrigeration and desalination equipments and in the handling of liquefied gases.

In single-phase fluids the phenomena of critical flow is well-understood and can be readily computed once the initial stagnation conditions and the geometry of the flow channel are given. However, while in single-phase flow the critical fluid velocity equals the local isentropic sound velocity, the critical velocity in two-phase flow is far more complicated to predict. This is due mainly to the interphase mechanical (slip) and thermal nonequilibrium resulting from the rapid expansion of the fluid through the break.

The main physical and mathematical aspects of the problem of two-phase critical flow have been discussed in several review articles (Bouré 1977; Jones & Saha 1977; Ardron & Furness 1976; Wallis 1980; Isbin 1980; Giot 1981; Elias & Lellouche 1993). A comprehensive review and discussion of the analytical models and key experimental results has also been compiled by D'Auria & Vigni (1980) and by Abdollahian *et al.* (1980, 1982), with about 250 references cited. The complexity of the thermodynamic phenomena taking place during a blowdown process resulted in many studies which simplified the evaluation of some of the transport phenomena taking place at the interface between the liquid and vapor bubbles in the flow. For instance, in the homogeneous equilibrium model (HEM) the interphase relative motion is neglected and a thermodynamic equilibrium is assumed. The HEM provides good results for the critical flow rate in long tubes where there is sufficient time for the two phases to reach equilibrium. The model, however, underpredicts the critical flow rate in short tubes where thermal and mechanical nonequilibrium effects must be considered as functions of the flow regime.

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In the last two decades a new approach has evolved based on a two-fluid representation of the flow. This representation, while allowing a detailed description of the interphase nonequilibrium phenomena, requires a great deal of information to complete the model formulation, such as constitutive relations to describe the interphase heat, mass and momentum transfer. Since this information cannot be obtained directly from measurements, critical flow models utilizing the conservation and balance equations of each phase generally differ in their treatment of the mechanical and thermal nonequilibrium between the phases. The interfacial momentum transfer (slip) is represented either by the drift-flux approximation, as in Kaizerman *et al.* (1983) and Elias & Chambré (1984), or by using a set of semiempirical correlations which determine the interphase drag forces (Ardron 1978; Richter 1981; Dobran 1987; Sami & Duong 1989).

Various levels of approximation are also used to estimate the heat and mass transfer rate at the liquid–vapor interface. For instance, Richter (1981) considers interphase conduction and convection heat transfer in a uniformly superheated liquid phase, while Ardron (1978) neglects the convection term and the bubble sphericity to derive an asymptotic solution of the bubble growth equation in a linearly varying temperature field. Lee & Schrock (1990) represented the initial degree of nonequilibrium by a modification of the Alamgir & Lienhard (1981) pressure undershoot correlation. A derivative-dependent exchange of mass and energy at the liquid–vapor interface is used in Sami & Duong (1989).

The rate of evaporation depends on the number density of vapor nuclei in the fluid. This parameter is difficult to measure and is typically considered either as a constant (Richter 1981; Dobran 1987) or is calculated by the homogeneous nucleation theory (Ardron 1978; Elias & Chambré 1984). In the latter case, the resulting number density of bubbles varies along the pipe.

The present work introduces a nonequilibrium two-fluid critical flow model, concentrating on the important mechanisms of heat and mass transfer to the vapor bubbles forming in the flow. A closed-form relation is derived to predict the bubbles rate of growth by conduction and convection in a superheated liquid, assuming an exponential pressure decay along the flow pipe. To close the model equations, an empirical correlation is derived which relates the number density of bubbles in the flow to the pipe's length-to-diameter ratio. With these improvements, the proposed model is shown to be applicable over a large range of stagnation conditions and flow path geometries.

2. PHYSICAL DESCRIPTION OF THE MODEL

The problem considered is that of a compressed fluid expanding from a large vessel through an open pipe. The model consists of five conservation equations that include mass and momentum equations for each phase and a combined energy equation for the two-phase mixture. A separate equation is introduced to relate the void spatial distribution to the growth rate of the vapor bubbles along the flow path.

The main assumptions underlying the model are: one-dimensional flow—although observations have indicated two-dimensional effects in two-phase critical flow (Edwards 1968), these phenomena have not yet gained a solid theoretical explanation. Next, the process is taken to be at steady-state. This is a commonly accepted assumption since transient effects during a blowdown of a large vessel are typically small. The flow is further assumed to be adiabatic. The only mode of heat transfer considered in the model is the internal exchange of heat between the phases during the formation and growth of vapor bubbles in the flow. At a given cross section along the pipe the vapor bubbles are considered to have equal sizes and to be uniformly distributed. Local pressure losses are assumed to take place adiabatically. The liquid temperature is supposed to remain constant at the entrance to the flow channel, while the vapor (when present) is assumed to be saturated with respect to the local pressure. With these assumptions the present model governing equations are (Wallis 1969):

vapor phase continuity,

$$\frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}z} = \frac{1}{\rho_G}\frac{\mathrm{d}\rho_G}{\mathrm{d}z} + \frac{1}{u_G}\frac{\mathrm{d}u_G}{\mathrm{d}z} + \frac{1}{\epsilon}\frac{\mathrm{d}\epsilon}{\mathrm{d}z} + \frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}z}; \qquad [1]$$

liquid phase continuity,

$$-\frac{1}{1-x}\frac{dx}{dz}=\frac{1}{\rho_L}\frac{d\rho_L}{dz}+\frac{1}{u_L}\frac{du_L}{dz}-\frac{1}{1-\epsilon}\frac{d\epsilon}{dz}+\frac{1}{A}\frac{dA}{dz}; \quad [2]$$

vapor phase momentum,

$$xG\frac{du_G}{dz}=-\epsilon\frac{dP}{dz}-F_{LG}-F_{wG}-\eta(u_G-u_L)G\frac{dx}{dz}-g\rho_G\epsilon\cos\theta; \quad [3]$$

liquid phase momentum,

$$(1-x)G\frac{du_L}{dz}=-(1-\epsilon)\frac{dP}{dz}+F_{LG}-F_{wL}+(1-\eta)(u_G-u_L)G\frac{dx}{dz}-g\rho_L(1-\epsilon)\cos\theta; \quad [4]$$

mixture energy,

$$[h_G-h_L+0.5(u_G^2-u_L^2)]\frac{dx}{dz}+x\left[\frac{dh_G}{dz}+u_G\frac{du_G}{dz}\right]+(1-x)\left[\frac{dh_L}{dz}+u_L\frac{du_L}{dz}\right]+g\cos\theta=0; \quad [5]$$

and

void fraction distribution,

$$\frac{d\epsilon}{dz}=4\pi R^2N\frac{dR}{dz}. \quad [6]$$

Where z is an axial coordinate and the subscripts G and L denote the vapor and liquid phase, respectively. ϵ, x and ρ are the void fraction, flow quality and density, respectively, A is the pipe cross-sectional area, F_{LG} is the drag forces at the liquid–vapor interface and F_{wG} and F_{wL} are the drag forces per unit volume exerted by the tube wall on the vapor and liquid, respectively. G is the total mass flux, h is enthalpy, P is the system pressure and u is velocity. η is a momentum distribution coefficient and θ is the pipe inclination angle. The vapor bubbles are characterized by their number density, N, and radius, R.

The solution of the model equations, [1]–[6], for the six state variables (h_L, u_G, u_L, x, P and ϵ) requires constitutive relations for F_{LG}, F_{wL}, F_{wG}, η and dR/dz. The thermodynamic properties $\rho_G(P)$, $h_G(P)$, $\rho_L(P, h_L)$ and their derivatives with respect to P and h_L, are calculated using the Haar *et al.* (1984) tables for saturated and superheated steam and water. The pipe geometry and orientation are considered to be known such that $A = A(z)$. The total mass flux, G, is treated as a parameter and the bubbles number density, N, is determined by a newly developed empirical correlation, as discussed later. The value of the momentum distribution coefficient, η, in the range 0 to 1, has been found to have only minor influence on the predictions of the pressure drop. $\eta = 0.5$ is used in this study. The interfacial force, F_{LG}, is modeled as follows (Richter 1981):

$$F_{LG}=2\frac{C_{fi}}{D_{pi}}\sqrt{\epsilon}\rho_G(u_G-u_L)|u_G-u_L|+C\epsilon\rho_L u_G\frac{d}{dz}(u_G-u_L), \quad [7]$$

where D_{pi} is the pipe diameter and C is a virtual mass coefficient, taken as $C = 0.5$ for bubbly flow ($\epsilon < 0.2$) and $C = 0$ otherwise. The interfacial friction factor, C_{fi}, is given for bubbly and annular ($\epsilon > 0.8$) flow as

$$C_{fi}=\begin{cases}C_D\sqrt{\epsilon}(1-\epsilon)^{-1.7}\dfrac{\rho_L}{\rho_G}\dfrac{D_{pi}}{2R} & \text{bubbly}\\ 0.005[1+75(1-\epsilon)] & \text{annular}\end{cases}. \quad [8]$$

In the intermediate range, between bubbly flow and annular flow, the interfacial friction factor is interpolated linearly with void fraction, ϵ, between the two values given in [8]. The drag coefficient for a single bubble, C_D, in [8] depends on the bubble Reynolds number, Re:

$$C_D=\begin{cases}\dfrac{24}{Re}(1+0.15\,Re^{0.687}) & Re<1000\\ 0.44 & Re>1000,\end{cases} \quad [9]$$

where Re is defined as

$$\mathrm{Re} = \frac{2\rho_L R(1-\epsilon)|u_G - u_L|}{\mu_L}. \quad [10]$$

The wall vapor friction term, F_{wG}, in the momentum equation [3] is neglected and F_{wL}, is modeled (Chisholm 1973) as

$$F_{wL} = -[1 + (\Gamma^2 - 1)][Bx^{(2-n)/2}(1-x)^{(2-n)/2} + x^{2-n}], \quad [11]$$

where $n = 0.25$ and Γ is a physical property coefficient defined by the square root of the ratio between the pressure gradient due to friction if the total mixture flows as vapor only, ΔP_{GO}, and that if the total mixture flows as liquid only, ΔP_{LO}:

$$\Gamma = \left(\frac{\Delta P_{GO}}{\Delta P_{LO}}\right)^{0.5} = \frac{f_{GO}}{f_{LO}}\frac{\rho_L}{\rho_G}; \quad [12]$$

f_{GO} and f_{LO} are the single-phase friction coefficients for the vapor and liquid, respectively.

The coefficient B in [11] is given by

$$B = \frac{C\Gamma - 2^{2-n} + 2}{\Gamma^2 - 1}, \quad [13]$$

where

$$C = \frac{u_L}{u_G}\sqrt{\frac{\rho_L}{\rho_G}}\left(1 + \frac{u_G^2}{u_L^2}\frac{\rho_G}{\rho_L}\right). \quad [14]$$

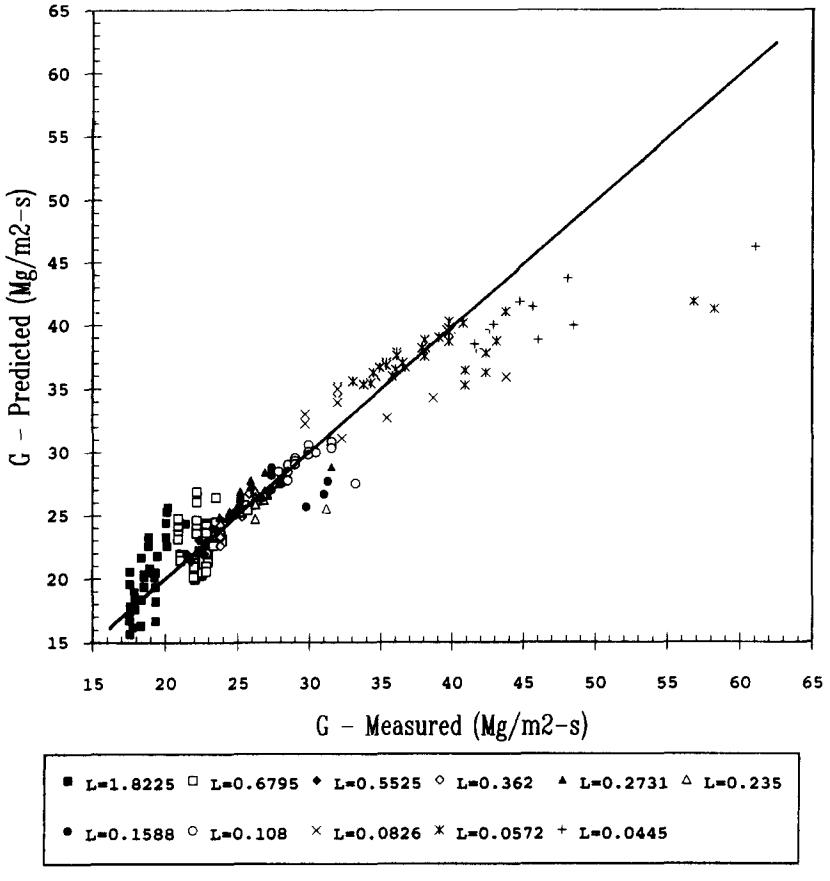

Figure 1. Comparison of the present theoretical predictions with Sozzi & Sutherland's (1975) rounded entrance nozzle 2, with saturated and two-phase mixture stagnation conditions.

To close the model set of equations, a constitutive relation was developed (Dagan 1989) relying on Olek *et al.* (1990), which predicts the bubble growth rate along the pipe. The relation is based on a diffusion-controlled growth of a vapor bubble in a superheated liquid, assuming an exponential pressure decay along the flow pipe. From the Clausius–Clapeyron relation the bubble vapor temperature is given by a linear function of the form

$$T_s(t) = T_s(0) - at, \qquad [15]$$

where t is time. The resulting relation for the bubble radius is

$$\frac{dR}{dz} = \frac{K_L}{\rho_G h_{LG} u_G}\left\{\frac{T_L(0) - T_s(0) + at}{R} + \frac{T_L(0) - T_s(0) + 2at}{\sqrt{\alpha_D \pi t}} + \frac{0.6\,\mathrm{Re}^{0.5}\,\mathrm{Pr}^{0.33}[T_L(0) - T_s(0)]}{2R}\right\} - \frac{R}{3\rho_G}\frac{d\rho_G}{dz}, \qquad [16]$$

where $T_L(0)$ and $T_s(0)$ are the initial liquid and saturation temperatures, respectively, h_{LG} is the latent heat of evaporation, K_L is the liquid thermal conductivity, a is the rate of liquid temperature variation, α_D is the liquid thermal diffusivity and Pr is the Prandtl number. Equation [16] accounts for convection and conduction heat transfer and neglects the short initial phase of inertia-controlled bubble growth. To utilize [16], a time coordinate, $t(z)$, is defined as the time taken for a typical bubble to reach position z from some initial location where the flow is nearly stationary. The time satisfies the kinematic relation

$$\frac{dt}{dz} = \frac{1}{u_G}. \qquad [17]$$

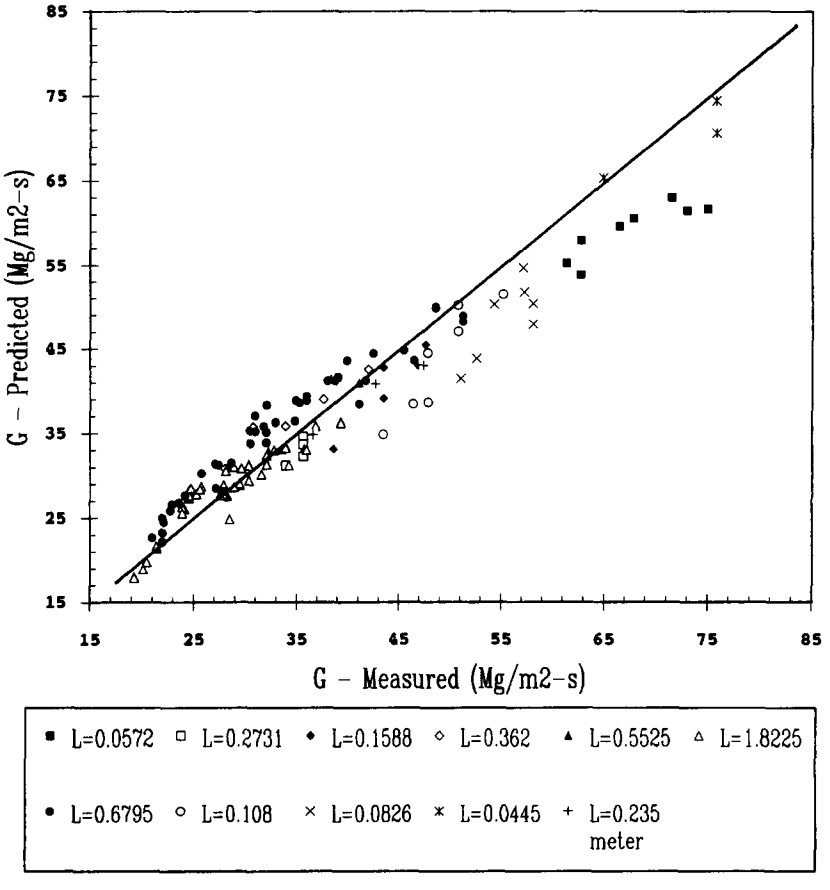

Figure 2. Comparison of the present theoretical predictions with Sozzi & Sutherland's (1975) rounded entrance nozzle 2, with subcooled stagnation conditions.

To predict the critical mass flux in a nondiverging flow channel of a given length, an exit mass flux, G is first postulated. Equations [1]–[6] and [16], [17] are then integrated to track the changing flow conditions along the channel, starting from the stagnation point at $z = 0$. The computation is terminated at a point, defined in this model as the choking plane, where the pressure gradient becomes arbitrarily large ($dp/dz > 50$ bar/mm). The postulated mass flux is changed iteratively until the choking plane coincides (within a prescribed error tolerance) with the pipe exit plane. The resulting mass flux is the critical mass flux for the specific experimental configuration.

It should be noted that in converging–diverging nozzles the choking point may occur anywhere along the diverging section (Bilicki *et al.* 1987; Bilicki & Kestin 1990; Lemonnier & Selmer-Olsen 1992). To apply the present model in converging–diverging nozzles, it may be assumed that the choking plane occurs at the throat, such that the model equations are integrated in the converging section of the nozzle only. This assumption should result in an overestimation of the critical mass flux. However, since typically the predicted location of the choking plane is very sensitive to small variations in the mass flux, the error in the predicted mass flux is expected to be small.

Numerical integration of the model equations was carried out, using the computer program Gear of the IMSL library. This algorithm was found suitable for this type of equations, which form a set of stiff differential equations having a wide range of time constants.

2.1. *Bubble number density correlation*

Previous studies have demonstrated the importance of the bubbles' number density, N, and their initial radii distribution on the predicted critical mass flux. Ardron (1978) and Elias & Chambré (1984) derived N from the homogeneous nucleation theory. Their model of bubble formation and growth implies a variable number density of bubbles along the flow conduit. Richter (1981) and

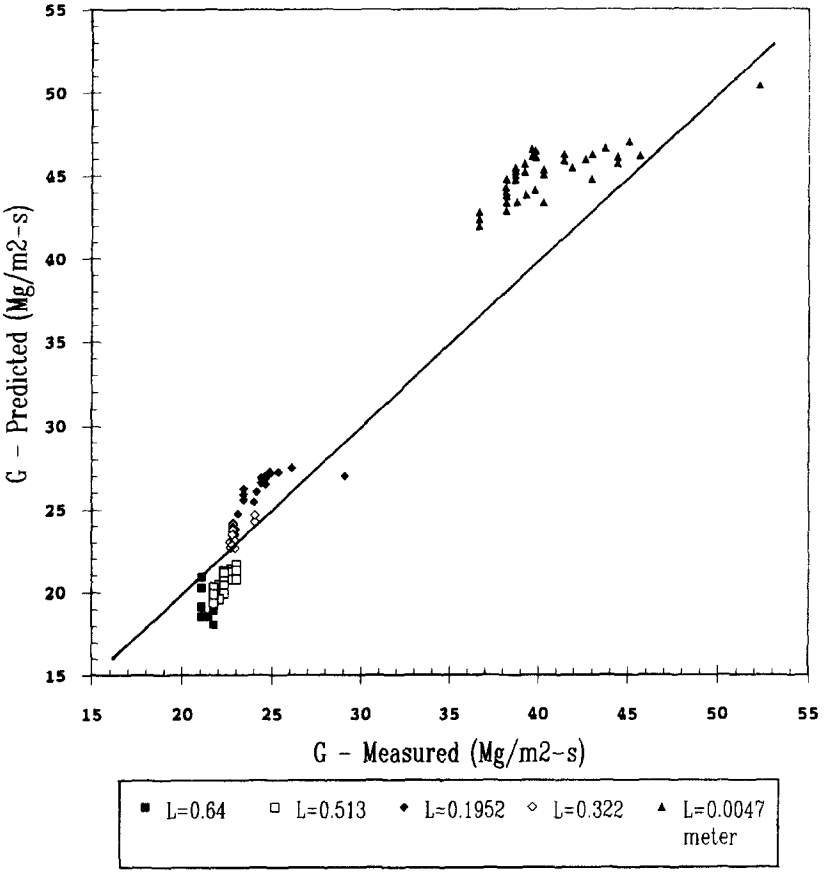

Figure 3. Comparison of the present model with Sozzi & Sutherland's (1975) sharp entrance nozzle data, with saturated and two-phase mixture stagnation conditions.

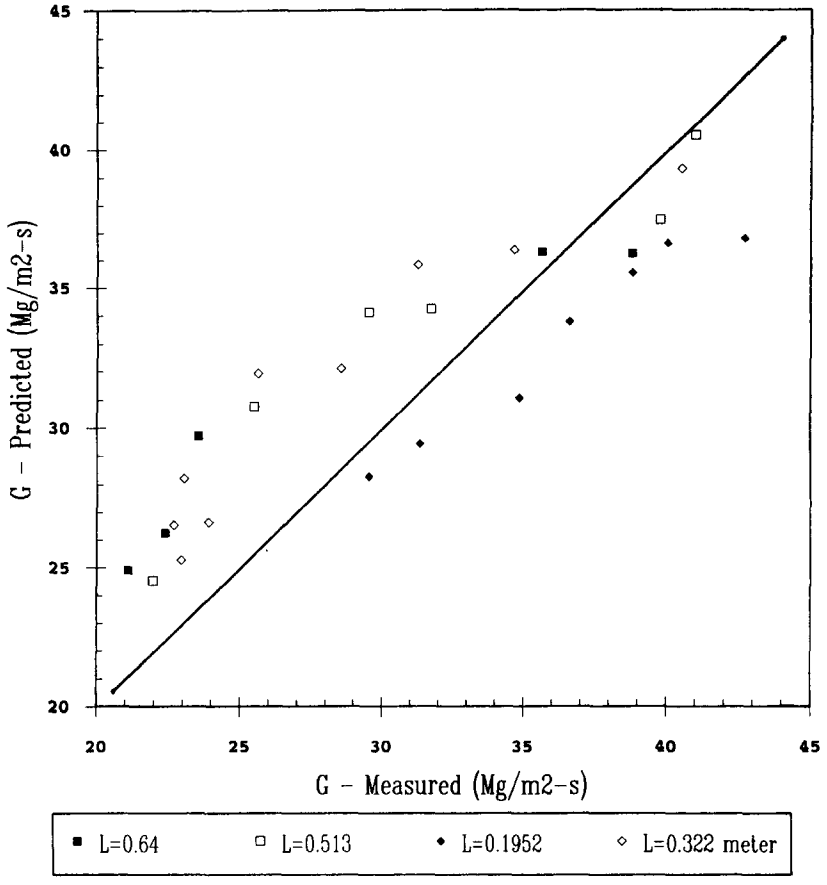

Figure 4. Comparison of the present model with Sozzi & Sutherland's (1975) sharp entrance nozzle data, with subcooled stagnation conditions.

Dobran (1987) used a constant value for N, independent of the stagnation conditions and the design parameters of the system. The present model shows that there is a substantial influence of N on the predicted critical mass flow rate. To simplify the computations, an empirical relation is derived which yields the bubble number density as a function of the channel's length-to-diameter ratio, L/D_{pi}. The correlation's parameters were obtained by fitting to the data of Sozzi & Sutherland (1975), but the same function was found to satisfy the other data considered in this study:

$$\ln N = \begin{cases} 24.0 + 2.0 \ln \dfrac{L}{D_{pi}}, & L/D_{pi} < 10 \\ 35.0 - 2.8 \ln \dfrac{L}{D_{pi}}, & L/D_{pi} > 10. \end{cases} \quad [18]$$

Equation [18] was tested in the range of $4 < L/D_{pi} < 150$. It demonstrates a correct physical trend by which the average density of vapor nuclei (and by [6], the rate of vapor generation) initially increases with L. As more vapor nuclei are introduced into the flow, the degree of thermal nonequilibrium is reduced and the rate of new bubbles formation is decreased. The location of the transition point in [18] at $L/D_{pi} = 10$ is in general agreement with other observations on the effect of L/D_{pi} on the critical flow rate. For instance, Henry (1970) postulated that vapor cannot be substantially generated in pipes with $L/D_{pi} < 12$. The effect of L/D_{pi} is also discussed by Jones & Saha (1977) and Ardron (1978), where it is observed that the discharge flow in pipes up to 10 diameters in length can be significantly larger than values obtained from homogeneous thermal equilibrium flow theory.

3. RESULTS AND DISCUSSIONS

The model was qualified in comparison with available high-pressure data on the adiabatic critical flow of water at subcooled, saturated or two-phase entry conditions. Predictions of the critical flow rate are found to be in favorable agreement with a wide range of experimental results in laboratory and commercial pipes. In figures 1 and 2 the model is compared with the data of Sozzi & Sutherland (1975) for saturated and subcooled stagnation conditions, respectively. The entrance geometry of nozzle 2 used in the experiments was approximated by a smooth elliptic contour which leads from the vessel into a straight 12.7 mm dia pipe. For subcooled entry conditions, bubbles are assumed to begin forming and grow when the system pressure drops to a level corresponding to more than 3°C of liquid superheat. Predictions of the present model are generally shown to agree with the measured data. Larger deviations between the measured and predicted results are observed mainly for short tubes in which the critical mass flux is higher due to the large degree of thermal nonequilibrium. However, the average deviation is lower, in most cases, than the reported experimental errors.

Figures 3 and 4 show the predicted critical flow rates for the sharp entrance nozzle configuration in the Sozzi & Sutherland (1975) experiments, having saturated and subcooled stagnation conditions, respectively. The results in this case are largely affected by the irreversible pressure drop at the entrance to the flow pipe. Although data scattering is more pronounced than observed in the predictions of the nozzle 2 data, model predictions follow the experimental trends. Deviations between measured and predicted results are generally $< 25\%$.

To investigate the predictive capability of the model for large-scale experiments, calculations were performed for a set of blowdown tests of the experimental program carried out at the

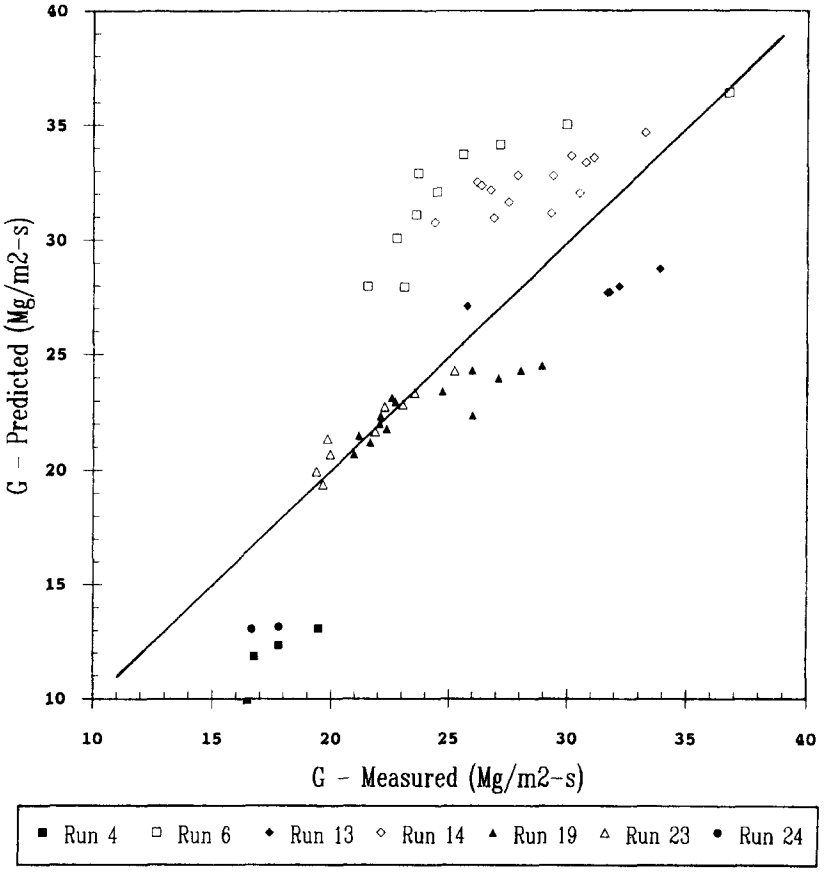

Figure 5. Comparison of the present theoretical predictions with the Marviken (1982) tests. The stagnation conditions are a saturated liquid and two-phase mixture.

Marviken (1982) test facility. The discharge nozzle considered for these predictions had a rounded entrance, followed by a 500 mm constant dia test section from 166 to 1809 mm in length. Since the upstream conditions changed very slowly during the 50–80 s blowdown period relative to the rapid (100 ms) adjustment of the flow in the test section, the data were treated in a quasi-steady manner, assuming constant stagnation conditions. Calculated results are compared with observed mass flow rates for initially saturated conditions in figure 5. In most cases the predicted results agree with the measured flow rates within the specified measurement accuracy. Runs 6 and 14 are overpredicted in some cases by up to 30%. Equation [11] used to predict the laboratory scale data of Sozzi & Sutherland (1975) was applied also to the prediction of the Marviken results in figure 5.

Marviken data with subcooled stagnation conditions are depicted in figure 6 for a range of pipe lengths and initial pressures. The maximum deviation between the measured and predicted flow rates is $\sim 25\%$ and the average is $< 10\%$.

4. SUMMARY AND CONCLUSIONS

A one-dimensional two-fluid model is presented for solving the steady-state critical two-phase flow phenomena. The model is validated against data measured in small- and large-scale test systems. It is shown that a satisfactory agreement with experimental data can be obtained by proper modeling of the heat and mass transfer processes between the two phases and the number density of bubbles in the flow. The model accounts for thermal and mechanical nonequilibrium in a low-quality flow using a conduction-controlled bubble growth. This mode of bubble growth may not be valid near the choking plane where, as pointed out by Olek *et al.* (1990), other parameters, such as bubble inertia, may be important.

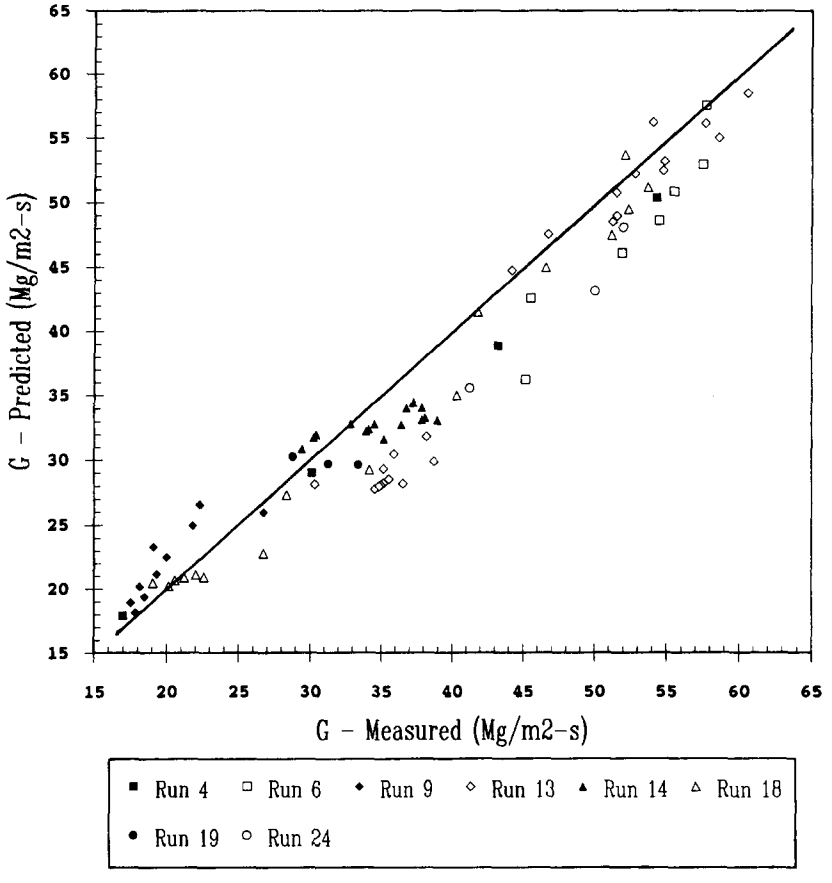

Figure 6. Comparison of the present theoretical predictions to the Marviken (1982) tests with subcooled stagnation conditions.

It is also recognized that the assumption of saturated vapor phase in a superheated liquid phase may not be realistic at high low qualities where vapor bubbles may coalesce to form a continuous phase. The model is, therefore, limited to low flow qualities in which direct thermal and momentum transfer between the bubbles may be neglected. Low quality critical flow is of great importance in the analysis of the initial phases of a depressurization transient in power reactors.

An empirical correlation is derived for the number density of bubbles in the pipe as a function of the pipe's length-to-diameter ratio. The correlation, simplifies the mathematical model and may provide a basic understanding of the important parameters influencing the predicted critical flow rate. Results suggest that the discharge flow in pipes up to 10 diameters in length can be significantly larger than values obtained from homogeneous thermal equilibrium flow theory.

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